Price, 25 cts. **OCTOBER, 1819.** No. IX.

J. T. MURDEN & CO. PRINT. P. AMBOY. | SOLD BY E. MURDEN, 3½ CHAMBER-ST. N. Y.

THE

SCIENTIFIC JOURNAL.

EDITED BY W. MARRAT, A. M.

Qu. 41. By Zero.

Show how to divide any cube number, as n^3, into 3 cubes.

Solution by the Proposer.

It is obvious that one ans. must be obtained by trial, and from that, others are to be deduced. It is manifest, also, that $\frac{8}{27} n^3$, $\frac{125}{216} n^3$ and $\frac{1}{8} n^3$, are 3 cubes, which ans. the qu. The sum of any two of these cubes can be divided into two other cubes by the common methods, and thence various ans. may be obtained. (Em. Al. prob. 76.) This can be accomplished by a more general method. Let a, b, c, be the roots of three other cubes, which are to be so found as to satisfy the conditions of the qu. Assume $a = \frac{2}{3} n - sv$, $b = \frac{5}{6} n - qv$, $c = \frac{1}{2} n - rv$, where s, q, and r, are arbitrary numbers, which are to be so taken that one of them at least may be negative, and v is a quantity to be determined. By the qu. $a^3 + b^3 + c^3 = n^3$, or $(\frac{2}{3} n - sv)^3 + (\frac{5}{6} n - qv)^3 + (\frac{1}{2} n - rv)^3 = n^3$; now, by expanding, $n^3 - 3 (\frac{4}{9} n^2 s + \frac{25}{36} n^2 q + \frac{1}{4} n^2 r) v + 3 (\frac{2}{3} ns^2 + \frac{5}{6} nq^2 + \frac{1}{2} nr^2) v^2 - s^3 + q^3 + r^3) v^3 = n^3$, or $- 3v n^2 (\frac{4}{9} s + \frac{25}{36} q + \frac{1}{4} r) + 3 nv^2 (\frac{2}{3} s^2 + \frac{5}{6} q^2 + \frac{1}{2} r^2) - v^3 (s^3 + q^3 + r^3) = 0$. Assume $\frac{4}{9} s + \frac{25}{36} q + \frac{1}{4} r = 0$, then will $q = - \frac{16s + 9r}{25}$, and $v = \frac{3n (\frac{2}{3} s^2 + \frac{5}{6} q^2 + \frac{1}{2} r^2)}{s^3 + q^3 + r^3} = \frac{4 s^2 + 5 q^2 + 3r^2}{2 (s^3 + q^3 + r^3)} \times n$. If now we assume r and s, q will be determined, and thence v will be known, and from thence

a, b, and c, may also be found. *Cor.* The same method will serve for the *qu.* by Analyticus in the globe, where it is proposed to divide the number 5 into four cubes; for after a few trials we find that $\frac{125}{27}$, $\frac{8}{27}$, $\frac{1}{27}$, $\frac{1}{27}$ are four cubes which satisfy the conditions; where we must substitute $\frac{5}{3}$ in the above expression for a, instead of $\frac{2}{3}n$; $\frac{2}{3}$ for $\frac{5}{6}n$, in that for b; and $\frac{1}{3}$ for $\frac{1}{2}n$, in that for c; we must also assume $d = \frac{1}{3} - vx$ for the root of the 4. cube, and then proceed as above. This qu. was also ans. by Mr. Phillips and Y.

Qu. 50. *By Galilei Galileo.*

Given, the base CB the alt. BA, and length AC, of an inclined plane; to find in AC a part = AB, through which a body would descend in the same time that another body would fall freely from rest at A through the alt. AB ?

1. *Solution by Y., of New-Haven.*

Let a be the perp. alt., s, the nat. sine of the plane's elevation, and x the dist. from the top of the plane to the uppermost point of the req. line; then $x + a$ will be the dist. to the lowest point, and by mechanics, time through a : time through x

$:: \sqrt{a} : \sqrt{\frac{x}{s}}$; and time through a : time through $a + x$ ::

$\sqrt{a} : \sqrt{\frac{a+x}{s}}$; and by division, time through a : time through

$(a + x) - x :: \sqrt{a} : \sqrt{\left(\frac{(a+x) - x}{s}\right)}$, but, by the conditions of the qu. the two first terms are equal, we have theref.

$$\sqrt{a} = \sqrt{\left(\frac{(a+x) - x}{s}\right)} \text{ and } x = (a - sa)^2 \div 4as.$$

2. *Solution by Philosophus, New-York.*

Put the time down the perp. alt. which is given, because the alt. is given, $= t$, $x =$ time from the top of the plane to the upper end of the line, which is $=$ to the perp., $e =$ sine of the plane's elevation, and $g = 16\frac{1}{12}$ feet, then $\frac{1}{2}gt^2 =$ perp. alt., which by the qu. is equal to $\frac{1}{2}ge\,((x + t)^2 - x^2;)$ that is $2ext + et^2 = t^2$, or $2ex = t - et$, ergo, $x = \frac{t - et}{2e}$. This qu. was also ans. by A. B.; Crito; Mr. Phillips, and Zero.

Qu. 51. *By Y.*

Rectify the curve whose equation is $y = \sin x$, by means of an elliptic arc; and find the superficies generated by a revolution of the curve round its axis, in finite terms?

Solution by the Proposer.

Conceive the curve whose ordinate is equal to the sine of its abscissa (rad. $= a$) to be applied to the surface of a cylinder, the rad. of whose base is a, in such a manner that the two extremities of the curve may be in opposite points of the base of the cylinder. Join these points by a straight line, which will be the diam. of the base. It is evident that any ordinate of the curve will be equal to a perp. drawn from its lowest point to this diam. for each will be the sine of the same arc. It is hence easily shown that all the points in the curve will lie in the same plane, and will constitute a semi ellipse, of which the conjugate is $= a$, and transverse $= a\sqrt{2}$. Every two corresponding portions of the ellipse, and of the figure of sines, will also be equal. In the same manner the length of the curve whose equ. is $y = m \sin. x$, will be found to be equal to that of a semi-ellipse whose conjugate axis is $= a$ and transverse $= a \sqrt{(1 + m^2)}$. If the analytical method be preferred, the differential of the length of the one may readily be transformed into that of the other of these curves; hence, as they begin together, the integrals must be equal. The general expression for the differential of the superficies produced by the revolution of a curve is $2pdy \sqrt{(dx^2 + dy^2)}$ but in the curve proposed, $dx = ady \div \sqrt{(a^2 - y^2)}$, hence by substitution $ds = 2py\, dy. \sqrt{\dfrac{2a^2 - y^2}{a \quad - y}}$. Put $v = \sqrt{(a^2 - y^2)}$, and the expression becomes $-2pdv \sqrt{(a^2 + v^2)}$ the integral of which is $-pv \sqrt{(a^2 + v^2)} - pa^2.\, h.\, l.\, (v + \sqrt{(a^2 + v^2)}$ which corrected by making $v = a$ (y and consequently $s = o$) we have $s = pa^2.\, h.\, l. \dfrac{a + a\sqrt{2}}{v + \sqrt{(a^2 + v^2)}} + pa^2\sqrt{2} - pv\sqrt{(a^2 + v^2)}$. When $y = a$, $v = o$; so that by reduction we have, $pa^2 (h.\, l.\, (1 + \sqrt{2}) + \sqrt{2})$; which is half the superficies of the entire solid.—This qu. was ans. also by Philosophus, Professor Crozet, and Zero.

Qu. 52. *By Mr. W. Marrat, New-York.*

Some writers on Navigation assert that the principles on

which *plane sailing* is founded are erroneous, the earth being considered as a globe; is this assertion true or false?

Solution by the Proposer.

The plane chart is erroneous, and it is founded on erroneous principles. It was an easy matter to assimilate the idea of *plane sailing* with that of the *plane chart*, and without particularly examining the subject, it would appear that both were founded on the same principles. Some writers have taken the meridional distance for departure, and on this supposition plane sailing could not be true. The mistake arose no doubt from not forming a just idea of departure, which is not a single line, but made up of all the indefinitely small lines on the globe, made on an oblique rhumb, and measured on the parallels of latitude crossed—this is demonstrated by Emerson, and some other writers, and the *principles* of plane sailing are thus proved to be *true*.—Solutions were also given to the same effect by Mr. J. Phillips; Y; and Zero.

QU 53. *By Mr. J. Campbell, Teacher, New-York.*

Given, $x^x = a$, to find the value of x^x?

Solution by Professor Adrain.

Take the log. of the given equation $x^x = a$, and we have $xlx = la$, of which the logarithm is $lx + llx = lla$: now put $lx = y$, $lla = b$, and we have $y + ly = b$. This equation may be easily solved by double position, or any of the usual methods of approximation, as is fully exemplified in the 2d No. of the *Analyst*.

When a is less than unity this solution fails, because the logarithm of a is negative, and the log. of la is imaginary. In such cases, put $x = \frac{1}{y}$, and $a = \frac{1}{b}$ and by substitution the equation $x^x = a$ becomes $b^y = y$. This equation, by taking the log. twice, becomes, first, $y.lb = ly$, and again, $ly + llb = lly$, which, by putting $ly = z$ and $llb = c$, becomes $z + c = lz$, an equation easily resolved by approximation. In resolving the equation $y + ly = b$, we may remove the negative parts of the logarithms, and even the decimals, by assuming $y = \frac{z}{1000, \&c.}$: if $y = \frac{z}{1000000}$ the equation $y + ly = b$ is converted into $z + 1000000\, lz = 1000000\,(6 + b.)$ For example, let $x^x = 123456789$. In this case, $b = 0.9080298$, and

therefore $z + 1000000\, lz = 6908030$. To find z, we have only to add the log. removing the decimal point 6 places to the right. This addition of a number and its log. may be done almost by inspection on Fol. 173. Hut. log. we thus obtain $z = 9365150$, whence $y = .9365150$, and $x = 8.640025$.

Another Solution.—Having from the given equation, $x^x = a$, obtained $lx + llx = lla$, put $llx = y$ and by substitution putting $e = 2.7182818$, &c. we have $y + e^y = lla$, the logarithms used being hyperbolic. This equation resolved by the general formula of La Grange, gives $y = lla - la + (la)^2 - \frac{3}{1.2}(la)^3 + \frac{4^2}{1.2.3}(la)^4 - \frac{5^3}{1.2.3.4}(la)^5 + \&c.$ Or $lx = la - (la)^2 + \frac{3}{1.2}(la)^3 - \frac{4^2}{1.2.3}(la)^4 + \frac{5^3}{1.2.3.4}(la)^5 - \&c.$ This value of lx being a given function of a, may be denoted by fa, consequently $lx = fa$, and $x = e^{fa}$.

Another Solution.—Extract the x^{th} root of the equation $x^x = a$, and we have $x = \sqrt[x]{a}$; in the 2d member of this equation put for x its value $\sqrt[x]{a}$, and we have $x = \sqrt[\sqrt[x]{a}]{a}$, in which last inserting as before $\sqrt[x]{a}$ for x, we obtain $x = \sqrt[\sqrt[\sqrt[x]{a}]{a}]{a}$, and continuing the operation indefinitely, $x = \sqrt[\sqrt[\sqrt[\sqrt[\&c.]{a}]{a}]{a}]{a}$. Or thus: transform $x^x = a$ into $y = b^y$ by putting $a = \frac{1}{b}$ and $x = \frac{1}{y}$; then in the right side of the equation $y = b^y$ write for y its equal b^y, and we obtain $y = b^{b^y}$, which last, in like manner, becomes $y = b^{b^{b^y}}$, and continuing the operation indefinitely we have

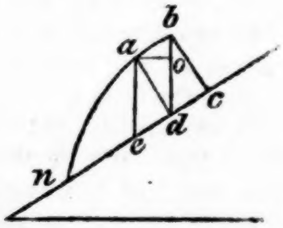
$y = b^{b^{b^{b^{\&c.}}}}$. Example 1. Let $x^x + 1 = 0$. Here $a = -1$, consequently, $b = -1$, and thus $y = (-1)^{(-1)^{(-1)^{\&c}}}$. But $(-1)^{-1} = \frac{1}{-1} = -1$, and therefore $(-1)^{(-1)^{(-1)^{\&c.}}} = -1$, therefore $y = -1$ and $x = -1$. Example 2. Let $x^x = 2$. In this case $b = \frac{1}{2}$ and computing by logarithms, we obtain $b^b = b'$, $b^{b'} = b''$ $b^{b''}$ b''' &c. In this way I found $x = 1.5596106$. The equation $x^x = a$ may also be resolved by the logarithmic curve, as will readily be perceived when we reduce the equation to the form $y + ly = b$, or to $y = b^y$. The intersections of a certain straight line with the logarithmic curve will give the values of y in these equations. We may also determine the values of x in the equation $x^x = a$ by the intersections of a logarithmic curve and an equilateral hyperbola. These constructions show when there is only one positive root in the equation, when there are two positive roots, and when the proposed equation has no positive root. There are besides roots among the discontinuous negative values of x, as in the example $x^x = \frac{1}{4}$, which cannot be satisfied by any positive value of x; the negative value is evidently $x = -\frac{1}{2}$. When a in the given equation $x^x = a$ is greater than unity, the equation has only one positive root; when a is greater than $e^{-\frac{1}{e}} = .6922$ but not greater than unity, the equation has always two positive roots, and when a is less than .6922, &c. the equation $x^x = a$ never has any positive root. In some cases we can obtain the imaginary roots as if $x^x = 0.207879$, &c. $= e^{-\frac{n}{2}}$ where $e = 3.7182818$, &c. and $n = 3.14159265$, &c. Here two of the values of x are $x = + \sqrt{-1}$. This qu. was ans. also by A. B.; Mr. Campbell, the proposer; Mr. J. Phillips; Y; and Zero.

Qu. 54. *By Mr. W. Marrat.*

A cylindrical vessel, full of water, and closed at the top, will just stand upon an inclined plane without falling over; if now a very small hole be made in the side, at the lowest point of the *upper* end of the vessel, to what dist. from the foot of the cylinder will the water spout upon the plane; the length of the vessel, and diameter of its base being 40 and 30 inches respectively?

Solution by the Proposer.

Let *cn* denote the inclined plane, and *abed* the cylindrical vessel, then, by the principles of dynamics, the water will spout from the vessel in a direction perp. to *ad*, and neglecting the resistance of the air, it will describe the parabola *an*, a tan. to the vertex of which is parallel to the inclined plane *nc*, and the parabola is ordinately applied to the plane. The centre of gravity of the cylinder being in the diagonal, *bd* is perp. to the horizon, *ae* is the abscissa, *en* the ordinate, and $4ob$ the parameter of the parabola *an*, also *abed* is a parallelogram.



Because $ad = 40$, and $ab = 30$, bd is found $= 50$, and by sim. triangles $ob = 18$, theref. $4ob =$ the parameter is $= 72$, and the abscissa $ae = 50$, and by the property of the parabola $ne^2 = 4ob \times ae$, or $ne = \sqrt{(72 \times 50)} = \sqrt{3600} = 60$, to which add $ed = ad = 30$, and we have $nd = 90$, for the distance required. The dist. nd is $= 3$ times the diameter of the base, and this will be the case, whatever may be the inclination of the plane. This solution is on the supposition that the water spouts from the vessel in a direction perp. to its side. It was ans. by Y. and Zero; the latter gentleman's solution agrees exactly with that given above, but Y. supposes that the water spouts out horizontally. In consequence of the above disagreement, recourse has been had to experiment, by which it has been proved that water spouts out *perp.* to the side of the vessel which contains it; the side being so thin, as not to give it a particular direction.

[*Solutions to the following questions must arrive before the* 1*st of December next.* *No. X. will appear on the* 1*st of January* 1820.]

Qu. 55. By Mr. J. Phillips, Teacher, Haerlem.

Given the alt. of a candlestick, having a circular top, and standing on a horizontal plane; to determine the height of the apex of the flame of a candle placed vertically in it, when the area of the top is to that of its shadow on the plane, in any given ratio?

Qu. 56. By Mr. Hostler.

Find the arc whose sine exceeds its versed sine by the greatest quantity possible—the arc being less than a quadrant.

Qu. 57. By Mr. J. Campbell, Teacher, New-York.

Given $10 - x = 10.h.l.\ 15 \div (5 + x)$; to find the value of x?

Qu. 58. *By Y.*

With a given quantity of metal, it is req. to construct a silver *cup* which shall hold the greatest possible quantity of fluid, and which, with a given thickness at the bottom (supposed inconsiderable with respect to the magnitude of the cup) shall be made of a thickness every where proportioned to the pressure of the contained fluid.

Qu. 59. By Zero.

Find the equ. of the curve whose axis is vertical, and ordinates horizontal, such, that the pressure of a body on different points of it, shall always vary as the ordinates belonging to those points.

Qu. 60. By Professor Crozet. W. Point.

The shadow cast by a regular hexagon, is traced on a floor, the hexagon being suspended vertically at a given height; find, 1. the position of the light; 2. the dimensions and position of the hexagon.

Qu. 61. By Mr. R. Taggart, Teacher, N. York.

Given the angle DAF, and in it the position of the point E, it is req. to draw through E a straight line DF, so that the perimeter ADF may be a minimum.

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Answer, by Y.

Let a be the height of the candlestick, and x the distance of the flame from the top of the stick, then, area of the top is to the area of its shadow :: $x^2 : (a+x)^2$. As this is a given ratio, let it be that of $u : u+1$; then, by division $x^2 : 2ax + a^2 :: u : 1$, and $x^2 - 2aux = ua^2$; which gives $x = a\,(u + \sqrt{(u^2+u)}\,)$. Mr. Campbell's answer is similar. It was answered also by Mr. Forest, Mr. Phillips, and Zero.

Qu. 56. By Mr. Hostler.

Find the arc whose sine exceeds its versed sine by the greatest quantity possible—the arc being less than a quadrant.

Answer, by Mr. W. Forest, New-York.

Put $x =$ cos. of the required arc, and $r =$ rad.; then $\sqrt{(r^2 - x^2)}$ is its sine, and $r - x$ its versed sine; and by the question $\sqrt{(r^2-x^2)} - (r-x)$, is to be a maximum, or $\frac{1}{2}\,(r^2-x^2)^{-\frac{1}{2}} \times -2xdx + dx = 0$, and by reduction $x = r \div \sqrt{2} =$ the cosine of 45°. Solutions were also given by Y. and Zero.

Qu. 57. By Mr. J. Campbell, Teacher, New-York.

Given $10 - x = 10.\ h.\ l.\ 15 \div (5+x)$; to find the value of x?

Answer, by the Proposer.

It appears from the equ. that x is nearly $=1$; but, by substitution, this is found to be too small; suppose it $=1.25$, then $\dfrac{15}{5+1.25}$ is $=2.4$, whose *h. l.* is ,87546, which, multiplied by 10 is $=8{,}7546$; but $10 - x = 10 - 1{,}25 = 8.75$; from whence we conclude that x is $= 1{,}25$, or $1\frac{1}{4}$, very nearly. If greater exactness be required, we may make use of the rule of trial and error. The equ. is taken from prob. 8, pa. 329, Hutton's Tracts, vol. 3, where the answer only is put down, and it was judged that it would be an easy exercise for learners. It was answered by Y. and Zero.

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posed inconsiderable with respect to the magnitude of the cup) shall be made of a thickness every where proportioned to the pressure of the contained fluid?

Solution, by Professor Adrain.

When the pressure, or thickness of the metal, is as any given function V of the rectangular co-ordinates x, y, z; x and y being horizontal, and z vertical, the formula to be made a maximum is, by the conditions of the question,

$$S\ (azdxdy + Vdxdy\ \surd(1 + z'^2 + z_{,}^2)$$

which, putting $U = az + V\ \surd(1 + z'^2 + z_{,}^2)$

becomes $\qquad S\ Udxdy.$

To resolve this, put $\dfrac{dU}{dz'} = U'$, $\dfrac{dU}{dz_{,}} = U_{,}$, and, by the general formula of La Grange, the required equation of the surface in partial differentials, will be

$$\frac{dU}{dz} - \left(\frac{dU'}{dx}\right) - \left(\frac{dU_{,}}{dy}\right) = 0.$$

Instead of pursuing this general equation, which would lead us into the greatest difficulties, I shall confine myself to a particular case, that may probably meet the views of the ingenious proposer.

When the pressure or thickness is as any function of the depth, the figure will be one of revolution, because there is nothing in the question to which the position of the horizontal axes can be referred. In this case, let $x =$ the abscissa of the generating curve, reckoning from the surface of the fluid on the axis of revolution, y and s, the corresponding ordinate and curve, and let X be a given function of x, measuring the pressure or thickness of metal at the depth x.

By the common method for problems of this nature, we must make a maximum of the formula,

$$S\ (ay^2dx + Xyds).$$

The variation of $ay^2dx + Xyds$, making y and dy constant, is

$$X'yds.\ \delta x + \left(ay^2 + Xy\frac{dx}{ds}\right).\delta dx.$$

But by the general rule, when the variation is

$$M\delta x + N\delta dx + P\delta d^2x + \text{\&c.}$$

the equation of the curve having the max. or min. is known to be $\qquad M - dN + d^2P - \text{\&c.} = 0:$

and in the present case,

$$M = X'yds,\ N = ay^2 + Xy\frac{dx}{ds},\ P = o,\ \text{\&c.}$$

therefore the required equation is

$$X'yds = d\left(ay^2 + Xy.\frac{dx}{ds}.\right),$$

which is a common differential equation of the 2d order.

Ex. 1. If the fluid be elastic, and of uniform pressure, $X = 1$, whence $X' = \frac{dX}{dx} = 0$, and the equation of the surface becomes $$o = d.\left(ay^2 + y\frac{dx}{ds}\right),$$
of which the integral is
$$C = ay^2 + y\frac{dx}{ds}.$$

But at the bottom of the cup, we may suppose $y = o$, therefore $C = o$, and in this case
$$ay + \frac{dx}{ds} = o \text{ ; this, by writing } -\frac{1}{r} \text{ for } a,$$
becomes $y = r\frac{dx}{ds}$, which shows that the required surface is spherical.

Ex. 2. If the fluid be of uniform density, and subjected to a constant gravity, the pressure or thickness will be as the depth. In this case, $X = x$, and $X' = 1$, and the required equation is
$$yds = d\left(ay^2 + xy\frac{dx}{ds}\right).$$

This equation does not relate to the circle, which is easily shown in the following manner:

If possible, let $y = r\frac{dx}{ds}$, be the common equation of a circle, coinciding with the curve, and since from $y = r\frac{dx}{ds}$, we have $yds = rdx$, and $\frac{y}{r} = \frac{dx}{ds}$, therefore the equation
$$yds = d\left(ay^2 + xy\frac{dx}{ds}\right)$$
becomes by substitution $$rdx = d\left(ay^2 + \frac{xy^2}{r}\right)$$
of which the integral is $$rx + C = ay^2 + \frac{xy^2}{r}.$$

Now if the two equations of the circle and of the curve were identical, or equivalent, the equation
$$rx + C = ay^2 + \frac{xy^2}{r}$$
must also belong to the same circle, which is impossible; since whatever value we assign to C, the equation belongs to

a line of the 3d order. In this case, therefore, the cup cannot be spherical.

Instead of making y and dy constant, we may with equal propriety make x and dx constant in the variation of the formula

$$ay^2\, dx + Xyds.$$

According to this hypothesis, the variation will be

$$(2\,aydx + Xds)\,\delta y + \left(Xy\,\frac{dy}{ds}\right)\delta dy:$$

and therefore, by the general rule, the equation of the required curve is

$$2\,aydx + Xds = d\,\left(Xy\,\frac{dy}{ds}\right).$$

This equation, though very different in appearance from what we found before, is, notwithstanding, perfectly equivalent, as will be evident from the following demonstration:

The two equations may be written thus—

$$X'yds - 2\,aydy = Xy.\, d\,\frac{dx}{ds} + \frac{dx}{ds}\,d\,(Xy),$$

$$Xds + 2\,aydx = Xy.\, d\,\frac{dy}{ds} + \frac{dy}{ds}\,d\,(Xy):$$

Multiply the former of these by $\frac{dx}{ds}$, and the latter by $\frac{dy}{ds}$, and they become

$$X'ydx - 2\,ay\,\frac{dxdy}{ds} = Xy.\,\frac{dx}{ds}\,d\,\frac{dx}{ds} + \left(\frac{dx}{ds}\right)^2.\, d\,(Xy)$$

$$Xdy + 2\,ay\,\frac{dxdy}{ds} = Xy.\,\frac{dy}{ds}\,d\,\frac{dy}{ds} + \left(\frac{dy}{ds}\right)^2.\, d\,(Xy).$$

Now let these equations be added, (observing that $\left(\frac{dx}{ds}\right)^2 + \left(\frac{dy}{ds}\right)^2 = 1$, and consequently $\frac{dx}{ds}\,d.\left(\frac{dx}{ds}\right) + \frac{dy}{ds}\,d.\,\frac{dy}{ds} = o,$) and we have

$$Xdy + yX'dx = d\,(Xy),$$

which being identical, shows that the two equations are perfectly equivalent.

When $X = 1$, the equations of the curve are

$$yds - 2\,aydy = d\,\left(xy\,\frac{dx}{ds}\right),$$

$$yds + 2\,aydx = d\,\left(xy\,\frac{dy}{ds}\right)$$

Let $S\ S'$ be the values of $\int yds$ and $\int xds$, which are the surfaces generated by the revolution of the curve about its axis

and its base diameter, omitting the common multiplicand 2π; and let $A = \int y dx =$ the area, then by integration

$$C + S - ay^2 = xy.\frac{dx}{ds}$$

$$C' + S' + 2\,aA = xy.\frac{dy}{ds},$$

which equivalent expressions, by the addition of their squares, produce

$$(C + S - ay^2)^2 + (C' + S' + 2\,aA)^2 = x^2 y^2,$$

which shows, in finite quantities, the nature of the curve. If we denote by φ the angle contained by the curve and ordinate, the nature of the curve may also be expressed by either of the following equations:

$$C + S - ay^2 = xy \sin.\varphi,$$
$$C' + S' + 2\,aA = xy \cos.\phi.$$

Qu. 57. By Zero.

Find the equation of the curve whose axis is vertical, and ordinates horizontal, such, that the pressure of a body on different points of it, shall always vary as the ordinates belonging to those points?

Answer, by the Proposer.

Let y denote any ordinate, x the abscissa, and z the corresponding length of the curve, then, if 1 denote the absolute weight of the body, $\frac{dy}{dz}$ will denote its pressure on the element dz of the curve; therefore if a denote the constant ratio of the pressure to the ordinate, we have, by the qu. $\frac{dy}{dz} = ay$, or $dy = aydz$, and $dy^2 = a^2y^2dz^2 = a^2y^2.(dy^2 + dx^2)$, or $dx = \frac{\surd(1 - a^2y^2)}{ay} \times dy$; and, integrating, $x + \frac{1}{2a}\,h.\,l.\left(\frac{1 - \surd(1 - a^2y^2)}{1 + \surd(1 - a^2y)}\right) + \frac{\surd(1 - a^2y^2)}{a} + C$, where C is a constant to be determined. If the origin of the abscissa be where $ay = 1$, or $y = \frac{1}{a}$, then $C = o$; hence we shall have $x = \frac{1}{2a}\,h.\,l.\left(\frac{1 - \surd(1 - a^2y^2)}{1 + \surd(1 - a^2y^2)}\right) + \frac{\surd(1 - a^2y^2)}{a} = h.\,l.\left(\frac{1 - \surd(1 - a^2y^2)}{ay}\right)^{\frac{1}{a}} + \frac{\surd(1 - a^2y^2)}{a}$ for the equation of the curve.

Cor. 1. The length of the curve, $z = \frac{1}{a}, h.\,l.\,ay = h.\,l.\,(ay)^{\frac{1}{a}}$.

Cor. 2. Its tangent is constant; for since $\frac{dy}{dz} = ay$, we have $\frac{ydz}{dy} = \frac{1}{a}$; but $\frac{ydz}{dy} =$ its tan. $= \frac{1}{a}$, which is constant, because a is a given quantity.

Remark. This curve agrees in kind with the curve described by the ball disturbed in qu. G.; for the lines joining the centres of the two balls is constantly 10 inches, and the ball disturbed constantly moves in the direction of the line joining the centres of the balls; but it moves also at any point, in the direction of a tan. to the curve at that point; then if the line joining the centres of the balls at any point, coincides with the tan. to the curve at that point, where the line of the abscissas is the line in which the ball that is not disturbed moves. The qu. then is, to find the length of the curve whose tan. is always 10 inches, the curve being reckoned from the point where the ordinate = sine of *one* minute to rad. 10. Put $s =$ sine of $1'$ to rad. 1. then $\frac{ydz}{dy} = 10$ or dz. $= \frac{10dy}{y}$, whence $z = 10 \times h.\ l.\ \left(\frac{10s}{y}\right)$, the curve being supposed to commence from the point where the ball is placed after it is moved one min. from its curve, and the whole curve described from the aforesaid point to where the tan. becomes perp. to the line of the abscissa $= 10 \times (h.\ l.\ s.)$, then the tan., if the balls continue their motion, will lie the contrary way to what it did at first, and the ball which before went first, will now follow the other, and continually approach the line of the abscissa without its centre ever actually arriving at it; therefore the line of the abscissa is the assypmtote of the part of the curve last described. Y. also answered this question, and remarked the coincidence of the two curves—he also deduced a great number of corollaries which are extremely curious, and some of them truly elegant; we intend to publish them as soon as we can find room. It may be observed, to students, that both these curves belong to a species of curve called the *Tractrix*, many elegant properties of which may be seen in a memoir written by M. Baumé, and published in the *Memoirs of the Academy of Sciences* at Paris.

Qu. 60. By Professor Crozet, W. Point.

The shadow cast by a regular hexagon, is traced on a floor, the hexagon being suspended vertically at a given height; find, 1, the position of the light; 2, the dimensions and position of the hexagon.

Answer, by Zero.

Conceive a circle to be described about the hexagon, (to be found,) and suppose that its vertical and horizontal diameters are drawn, and that tangents are drawn to the extremities of the horizontal diameter, and they will be perp. to the floor. Let also the opposite angles of the hexagon be joined by straight lines, and they will all cross at the centre of the circle. Let then two planes be drawn through the candle, so as to coincide with the vertical tangents, the planes will both be perp. to the floor, because the tangents are so (Playfair's Eu. 17. 2. Sup.;) hence, their common section, which passes through the candle, will be perp. to the floor, (18. 2. ut supra,) by which the candle will be orthographically projected upon the floor. Now this point can be found as follows: conceive straight lines drawn from the candle (before it is projected) to every point of the circle, and they will form a cone, whose vertex is at the candle. Now the plane of the floor, cutting the surface of this cone, will cut off some conic section, and if straight lines be drawn from the candle to all the angles of the hexagon in the circle, they will lie in the surface of the cone. If we extend them till they meet the floor, the points in which they meet it will be the projection of the angles of the hexagon, which will lie in the perimeter of the conic section, because the projecting lines all lie in the surface of the cone; and if the conic section be an ellipse, or circle, all the angles of the hexagon will be projected on the floor; otherwise not. Hence it appears that the figure traced upon the floor can be inscribed in some conic section. Again, it is manifest that the planes drawn through the candle, and the vertical tangents will cut the floor, in two straight lines, which will be the projections of those tangents; also, that those lines will touch the conic section. Further, if we draw straight lines from the candle to every point of the horizontal diameter of the circle, and extend them till they meet the floor, this diameter will be projected into a straight line parallel to itself, which will join the points of contact of the abovementioned tangents to the conic section; also, the centre of the circle will manifestly be projected into the middle of the projected horizontal diameter; and if we project all the diagonals of the hexagon, their projections will all pass through the middle of the projected horizontal diameter. If we draw tangents to the circle at the extremities of the vertical diameter, and project them, together with their vertical diameter, by drawing straight lines from the candle to every point in them, and extend them till they meet the floor, the projection of the horizontal tangents will be parallel to the horizontal diameter, and the projection of the vertical diameter will pass through their points of contact, and through the middle of the projected horizontal diameter, and

through the orthographic projection of the light upon the floor. Hence it is manifest that the projection of the horizontal diameter is a double ordinate in the conic section to the projection of the vertical diameter. The above is manifest from the principles of perspective.

(To be continued.)

Solutions to the following questions must arrive before the 1st of March, 1820. The next number will be published on the 1st of April.

Qu. 62. By Mr. Jephson.

Extract the square root of $4\,mn + 2\,(m^2 - n).\ \sqrt{(-1)}$?

Qu. 63. By Mr. M'Farlan.

In a triangle whose sides are a and b, and included angle $\frac{4}{3}$ a right angle, the square of the base is $= \dfrac{a^3 \backsim b^3}{a \backsim b}$: required the demonstration?

Qu. 64. By Mr. Hostler.

If a rhumb line be always inclined to the meridians in an angle of 60°, its length from the equator to the pole is equal to half that of a great circle of the sphere: the proof is required?

Qu. 65. By Y.

Every vulgar fraction, whose denominator contains any prime factor besides 2 and 5, not common to the numerator, is equivalent to a repeating decimal. In every other fraction, the division of the numerator by the denominator will terminate. Required the demonstration?

Qu. 66. By Y.

It is required to construct, geometrically, a trapezium, whose sides shall be equal to given lines, and which may be inscribed in a circle?

Qu. 67. By Zero.

It is required to draw the shortest line possible from one point to another, on the surface of a spheroid?